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Determination of Resonating Frequency of Thin Rectangular Flat Plates

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Abstract: **The development of a general computer program for analysis of free vibration of rectangular thin plates using polynomial functions is the focus of this study. A general polynomial shape function was first derived and then Ritz energy equation used to obtain an equation in terms of a non-dimensional parameter 'k', for the resonating frequency of a vibrating plate. Thereafter, Matlab programming language was used to develop an interactive computer program which requires the user to input the shape function and dimensions of each plate under consideration in order to obtain the resonating frequency. The validity of the program was demonstrated by comparing the predicted resonating frequencies with those obtained by other relevant researchers. The percentage differences were minimal and insignificant. Thus, the developed program can be used for easy and quick free vibration analysis of rectangular plates.**

Keywords: **Determination, Matlab Programming, Polynomial Shape Function, Resonating Frequency, Rectangular Plate, Ritz Energy Equation, Free Vibration.**

Symbols

w = Deflection; w_{max} = Maximum Deflection;

U, V= Deflection parts in X- & Y- Directions for Non-dimensional Parameters

R or r = Non dimensional Parameter in X- direction and is equal X/a

Q or q = Non dimensional Parameter in Y- direction and is equal Y/b

a = dimension along X -direction; b = dimension along Y- direction

$w''^{R} = \frac{\partial^2 w}{\partial R^2}$; $w''^{Q} = \frac{\partial^2 w}{\partial Q^2}$; $w''^{RQ} = \frac{\partial^2 w}{\partial R\, \partial Q}$; $k''^{R} = \frac{\partial^2 k}{\partial R^2}$; $k''^{Q} = \frac{\partial^2 k}{\partial Q^2}$; $k''^{RQ} = \frac{\partial^2 k}{\partial R\, \partial Q}$

ρ = Specific Gravity of plate material, h = plate thickness; D = flexural rigidity.

1. INTRODUCTION

Free vibration of rectangular plate can be studied by specifying the boundary conditions of a plate. Often times, undesirable excitations both internal and external, have been experienced by structural elements such as plates. The most important thing in the analysis of free vibration, is the resonating frequency, which is the value of externally induced vibrating frequency on the plate that causes it to resonate. [1], [2], [3], [4], [5], [6] and many other scholars have carried out studies on free vibration of plates using classical and approximate methods based on trigonometric shape function. And of most recent scholars like [7], [8], [9], [10], [11], did such analysis in a different way using polynomial shape functions. [12], [13] tried applying the used of programing method in CCCC plate analysis.

But, there is dearth of literature on the development of computer programs based on polynomial functions to ease the difficulties of the former approaches. Therefore, the purpose of the present study is to design a computer program for easy, quick and inexpensive free vibration analysis of rectangular plates.

2. POLYNOMIAL SHAPE FUNCTION.

The general shape function of thin rectangular plate, was derived by assuming a deflected shape function (w) in form of polynomial series which was truncated at the fifth term. This deflection was consider in X-Y directions using non-dimensional parameters R and Q (where R =X/a and Q = Y/b.

For R-direction, the deflected shape function, w^R , is given as Eqn (1)

$$w^R = a_0+a_1R+a_2R^2+a_3R^3+a_4R^4 \quad (1)$$

For Q-direction, the deflected shape function, w^Q , is as follows:

$$w^Q = b_0+b_1Q+b_2Q^2+b_3Q^3+b_4Q^4 \quad (2)$$

where a_0, a_1, a_2, a_3, and a_4, and b_0, b_1, b_2, b_3, and b_4 are numerical coefficients

Boundary Conditions:

Simply Supported Edge: $w = 0$; $d^2w/dR^2 = d^2w/dQ^2 = 0$

Clamped Edge: $w = 0$; $dw/dR = dw/dQ = 0$

Free Edge: $d^2w/dR^2 = d^2w/dQ^2 = 0$; $d^3w/dR^3 = d^3w/dQ^3 = 0$

Different boundary conditions for each plate to be considered were applied to Eqns (1) and (2). The deflected shape function, (w), for each plate was then obtained by multiply Eqns (1) and (2) which yields an equation in form of Eqn (3)

$$w = w^R*w^Q = A(c_1R^{f1}-d_1R^{m1}+e_1R^{n1})\ (c_2Q^{f2}-d_2Q^{m2}+e_2Q^{n2}) \quad (3)$$

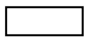
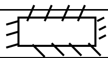
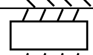
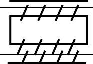
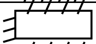
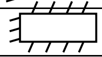
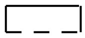
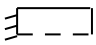
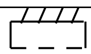
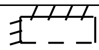
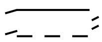
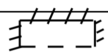
where c_1,c_2,f^1,f^2,d_1,d_2,m^1,m^2,e_1,e_2 and n^1,n^2 are coefficients. See TABLE 1 for polynomial shape functions for the 12 different rectangular plates; A = amplitude of deflection

$$\text{let } U = (c_1R^{f1}-d_1R^{m1}+e_1R^{n1}) \text{ and } V = (c_2Q^{f2}-d_2Q^{m2}+e_2Q^{n2}) \quad (4)$$

$$\text{Therefore, } w = A*U*V = Ak \quad (5)$$

The deflected shape functions derived for some plates are presented on TABLE 1

TABLE 1: Formulated Polynomial Shape Functions

S/N	EDGE TYPE	SHAPE	SHAPE FUNCTION 'W' W = AK
1	SSSS		$A(R-2R^3+R^4)(Q-2Q^3+Q^4)$
2	CCCC		$A(R^2-2R^3+R^4)(Q^2-2Q^3+Q^4)$
3	CSSS		$A(R-2R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
4	CSCS		$A(R-2R^3+R^4)(Q^2-2Q^3+Q^4)$
5	CCSS		$A(1.5R^2-2.5R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
6	CCCS		$A(1.5R^2-2.5R^3+R^4)(Q^2-2Q^3+Q^4)$
7	SSFS		$A(R-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
8	SCFS		$A(1.5R^2-2.5R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
9	CSFS		$A(R-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
10	CCFS		$A(1.5R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
11	SCFC		$A(R^2-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
12	CCFC		$A(R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$

= Fixed Edge = Simply Support Edge = Free Edge

3. APPLICATION OF RITZ ENERGY EQUATION

The total potential energy functional, Π, for free vibration of rectangular Isotropic plate using Ritz method as given by Ibearugbulem (2013), is as given in Eqn (6)

$$\Pi = \frac{D}{2b^2} \iint \left[\frac{1}{P^3} (w''^R)^2 + \frac{2}{P} (w''^{RQ})^2 + p(w''^Q)^2 \right] \partial R \partial Q - p\rho h \omega^2 b^2 \iint w^2 \, \partial R \partial Q \qquad (6)$$

where p = a/b is the aspect ratio; ω = resonating frequency.

Substituting Eqn (5) into Eqn (6), gives

$$\Pi = \frac{DA^2}{2b^2} \iint \left[\frac{1}{P^3} (k''^R)^2 + \frac{2}{P} (k''^{RQ})^2 + p(k''^Q)^2 \right] \partial R \partial Q - p\rho h \omega^2 b^2 A^2 \iint k^2 \, \partial R \partial Q \qquad (7)$$

Minimizing Eqn (7) and making ' ω^2' the subject of the formula, yields

$$\omega^2 = \frac{\int\int [\frac{1}{P^4}(k''^R)^2 + \frac{2}{P^2} (k''^{RQ})^2 + (k''^Q)^2] \, \partial R \, \partial Q}{\int\int k^2 \, \partial R \, \partial Q} * \frac{D}{\rho h b^4} \qquad (8)$$

$$\text{Let } f_p{}^2 = \frac{\int\int [\frac{1}{P^4}(k''^R)^2 + \frac{2}{P^2} (k''^{RQ})^2 + (k''^Q)^2] \, \partial R \, \partial Q}{\int\int k^2 \, \partial R \, \partial Q} \qquad (9)$$

Therefore, Eqn (8) becomes

$$\omega^2 = f_p{}^2 \, \frac{D}{\rho h b^4} \qquad (10)$$

$$\text{Hence, } \omega = \frac{f_p}{b^2} \sqrt{\frac{D}{\rho h}} \qquad (11)$$

where f_p is the coefficient of resonating frequency for aspect ratio of p = a/b.

The Eqn (8) or (11) is the general formula for determining the resonating frequency of thin isotropic rectangular plate with aspect ratio, p = a/b.

For aspect ratio s = b/a = 1/p (12)

Eqn (8) becomes

$$\omega^2 = \frac{\int\int [(k''^R)^2 + \frac{2}{S^2}(k''^{RQ})^2 + \frac{1}{S^4} (k''^Q)^2 \, \partial R \, \partial Q}{\int\int k^2 \, \partial R \, \partial Q} * \frac{D}{\rho h a^4} \qquad (13)$$

$$\text{Let } f_s{}^2 = \frac{\int\int [(k''^R)^2 + \frac{2}{S^2}(k''^{RQ})^2 + \frac{1}{S^4} (k''^Q)^2 \, \partial R \, \partial Q}{\int\int k^2 \, \partial R \, \partial Q} \qquad (14)$$

$$f_s = \left[\frac{\int\int [(k''^R)^2 + \frac{2}{S^2}(k''^{RQ})^2 + \frac{1}{S^4} (k''^Q)^2 \, \partial R \, \partial Q}{\int\int k^2 \, \partial R \, \partial Q} \right]^{1/2} \qquad (15)$$

Noting Eqns (15) and (15), Eqn (13) becomes

$$\omega^2 = f_s{}^2 \, \frac{D}{\rho h a^4} \qquad (16)$$

$$\text{Hence, } \omega = \frac{f_s}{a^2} \sqrt{\frac{D}{\rho h}} \qquad (17)$$

where f_s is the coefficient of resonating frequency with aspect ratio, s = a/b.

The Eqns (13) or (17) is the general expression for computing the resonating frequencies of thin isotropic rectangular plate with aspect ratio, s = b/a.

4. COMPUTER PROGRAMMING

A Matlab Program was designed for the analysis of free vibration of plates and presented in the Appendix. The values of the coefficients of the resonating frequency ‘fs’ obtained from the program with aspect ratio, s = b/a, are presented on TABLE 2. This program requires the designer to input the shape functions of any of the rectangular plate considered, some of which are presented in column 3 of table 1 into the program and follow other on screen responses. The user should be familiar with the usage of Matlab. The algorithm is as follows:

ALGORITHM

- ✓ Start
- ✓ Input the dimensions of plate, a, and, b; Poisson ratio, v; plate thickness, h; Young's modulus, E; and specific density of plate, ρ. Press 'Enter' for each variable you input.
- ✓ Calculate aspect ratio , s = b/a
- ✓ Calculate flexural rigidity, $D = Eh^3/12(1-v^2)$
- ✓ Input the U (i.e r terms of parameter, k) and V (i.e q terms of parameter, k).
- ✓ Calculate the coefficients of resonating frequency , $f = \frac{\int\int [(k''^{r})^2 + \frac{2}{s^2}(k''^{rq})^2 + \frac{1}{s^4}(k''^{R})^2]\,\partial r\,\partial q}{\int\int k^2\,\partial r\,\partial q}$
- ✓ Calculate f_1 in terms of b, then $f_1 = f/s^2$
- ✓ Calculate $f_2 = f_1/\pi^2$
- ✓ Calculate the resonating frequency,$F = \frac{f}{a^2}\sqrt{\frac{D}{\rho h}}$ where F = ω, and $f = f_s$
- ✓ End

5. RESULTS AND DISCUSSIONS

To check the accuracy of the values obtained from this program, comparison was made with the 'λ' or f_{1s} obtained from the present study and that of Ibearugbulem et al. (2014) in TABLE 3 for two of the plates CCSS and CCFS, it indicates that the percentage differences are all 0.00% for CCSS plate. This implies that the values from this program are the same with those from [11]. Also, comparing that of CCFC plate, the percentage differences were all less than 3% with an average of 2.317% which is negligible. It also, shows that the values of CCFC from the present study are slightly upper bound to those of [11].

TABLE 2: Coefficients 'f_s' of Resonating Frequency

Aspect Ratio S= b/a S	CCSS		CCFC	
	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$ f_S	$\omega = f_{1S}\frac{\pi^2}{a^2}\sqrt{\frac{D}{\rho h}}$ f_{1S}	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$ f_S	$\omega = f_{1S}\frac{\pi^2}{a^2}\sqrt{\frac{D}{\rho h}}$ f_{1S}
1.0	27.129	2.749	24.669	2.500
1.2	23.095	2.340	23.843	2.416
1.5	20.019	2.028	23.260	2.357
1.6	19.394	1.965	23.146	2.345
2.0	17.840	1.808	22.869	2.317

6. CONCLUSION

Results of this study has shown that the values from the present program for the two sample plates CCSS and CCFC are very close to those of earlier studies. Hence, we can conclude that this program is satisfactory, faster and easier for free vibration analysis of rectangular plates.

TABLE 3: Values of ' λ ' from present study with those of [11].

Aspect Ratio S = b/a S	Present study CCSS λ_1	Ibearugbulem et al. (2014) λ_2	% difference $100(\lambda_1 - \lambda_2)/\lambda_2$	Present study CCFC λ_3	Ibearugbulem et al. (2014) λ_4	% difference $100(\lambda_3 - \lambda_4)/\lambda_4$
1.0	2.749	2.749	0.00	2.500	2.441	2.417
1.2	2.340	2.340	0.00	2.416	2.374	1.769
1.5	2.028	2.028	0.00	2.357	2.329	1.202
1.6	1.965	1.965	0.00	2.345	2.321	1.034
2.0	1.808	1.808	0.00	2.317	2.302	0.652
Aver. %diff.			0.00			1.415

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APPENDIX

```
clear
%PROGRAM FOR DETERMINING RESONATING FREQUENCIES OF RECTANGULAR PLATES
v = input('Enter value of poission ratio v:');
a = input('Enter plate dimension  along x-axis -length- a(m):');
b = input('Enter plate dimension  along y-axis -width- b(m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
%Deflected shape function w = Ak; k = U*V
syms r q
U = input('Enter U:');
V = input('Enter V:');
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
```

```
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
Y6 = y2*z1;
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),5)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),5)
```